2

$A=\{1,3,5\}$
$B=\{3,4\}$
$C=\{5,6\}$

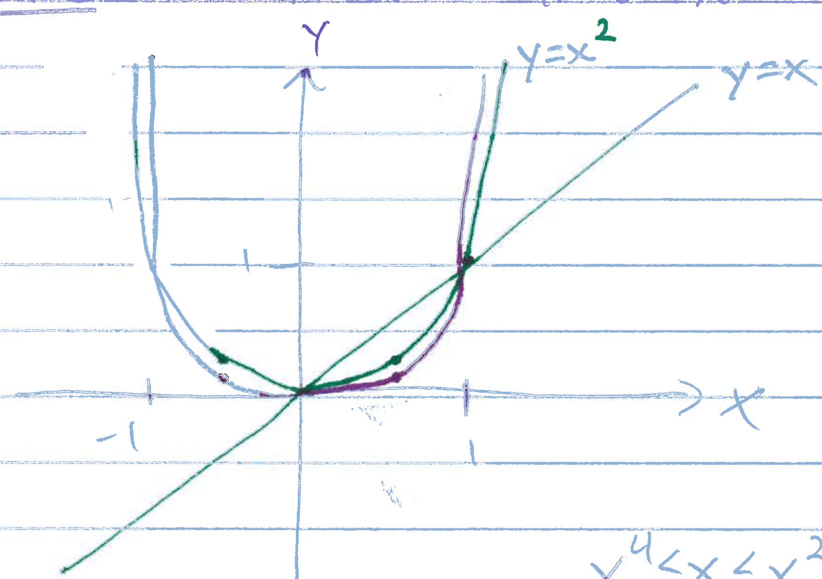
$B\cup C=\{3,4,5,6\}$
$A-(B\cup C)=\{1\}$
$A-B=\{1,5\}$
$A-C=\{1,3\}$
$(A-B)\cup(A-C)=\{1,3,5\}\neq A-(B\cup C)$

Disproving $\exists x\in S, P(x)$ Statements

To disprove this we have to prove

$$\forall x\in S, \sim P(x)$$

Conjecture $\exists x\in\mathbb{R}$ such that $x^4<x<x^2$.



$x^4<x<x^2$ is never true in the graph.

So, we have to show that $\forall x\in\mathbb{R}$, $x^4<x<x^2$ is false

③

Suppose there is an x such that $x^4 < x < x^2$.
If $x = 0$, we have $0 < 0 < 0$ which is false.
So we can assume $x \neq 0$. Since x^4 and x^2 are positive, we have that x must be positive.
Then dividing by x:

$$x^3 < 1 < x$$

$$\Rightarrow x^3 - 1 < 0 < x - 1 \quad (*)$$

$$\Rightarrow (x-1)(x^2+x+1) < 0 < x-1$$

$$\Rightarrow x^2+x+1 < 0 < 1 \quad ((*) \text{ says } x-1 \text{ is positive})$$

$\Rightarrow x^2+x+1 < 0$, however x^2+x+1 is positive, a contradiction.

Conjecture: $\exists$ prime numbers p, q such that $p - q = 97$

Assume this is true. Then p and q have opp. parity, i.e., one is even and one is odd. The even prime must be 2, so we have $q = 2$. (o/w $p - q < 0$)
Since $q = 2$, $p - q = p - 2 = 97 \Rightarrow p = 99$. But 99 is not prime. Contradiction.

①

Induction

Prop: The statements $S_1, S_2, S_3, S_4, \cdots$ are all true.

~~proof~~ ① Show S_1 is true (base case)
② Assume S_k is true for an arbitrary k (inductive hypothesis)
③ Show $S_k \Rightarrow S_{k+1}$ is true. (inductive step)

Prop: $\forall n \in \mathbb{N}$, $1+3+5+\cdots+(2n-1)=n^2$

Prop: For nonnegative integers n, $n^5 \equiv n \bmod 5$.

Prop: Let n be a nonnegative integer. Then

$$\sum_{i=0}^{n} i \cdot i! = (n+1)! - 1$$

Prop 10.1: Suppose $a_1, \cdots, a_n$ are n integers, $n \geq 2$. If p is prime and $p \mid a_1 a_2 \cdots a_n$, then $p \mid a_i$ for at least one a_i.